Name:____________________________ Date:_______________________

Math 10/11 Honors: Challenging 4 Sequences and Series Problems:

1. For each positive integer 'k', let S_k denote the increasing arithmetic sequence of integers whose first term is 1 and whose common difference is "k". For example, S_3 is the sequence 1, 4, 7, …. Find how many values of "k" does S_k contain the term 2005? (aime)

2. The terms of an arithmetic sequence add to 715. The first term of the sequence is increased by 1, the second term is increased by 3, the third terms is increased by 5, and in general, the kth term is increased by the kth odd positive integer. The terms of the new sequence add to 836. Find the sum of the first, last, and middle terms of the original sequence. (aime)

3. Find the sum of $2^3 - 1^3 + 4^3 - 3^3 + 6^3 - 5^3 + \ldots\ldots\ldots + 18^3 - 17^3$ (AMC)

4. The sum of the first 2011 terms of a geometric series is 200. The sum of the first 4022 terms of the same series is 380. Find the sum of the first 6033 terms of the series (aime)

5. Two distinct, real, infinite geometric series each have a sum of 1 and have the same second term. The third term of one of the series is 1/8, and the second term of both series can be written in the form $\frac{\sqrt{m}-n}{p}$, where "m", "n", and "p" are positive integers and "m" is not divisible by the square of any prime. Find $100m + 10n + p$.

6. Find the eight term of the sequence: 1440, 1716, 1848…… Where the terms in this sequence are formed by multiplying the corresponding terms of two arithmetic sequence. (aime)

7. In an increasing sequence of four positive integers, the first three terms form an arithmetic progression, the last three terms form a geometric progression, and the first and fourth terms differ by 30. Find the sum of the four terms (aime)

8. Find the smallest prime that is the fifth term of an increasing arithmetic sequence, all four preceding terms also being prime. (aime)

9. Let $N = 100^2 + 99^2 - 98^2 - 97^2 + 96^2 + ... + 4^2 + 3^2 - 2^2 - 1^1$, where the additions and subtractions alternate in pairs. Find the remainder when "N" is divided by 1000 (aime)

10. Dave rolls a fair six-sided die until a six appears for the first time. Independently, Linda rolls a fair six sided die until a six appears for the first time. Let "m" and "n" be relatively prime positive integers such that $\frac{m}{n}$ is the probability that the number of times Dave rolls his die is equal to or within one of the number of times Linda rolls her die. Find "m + n"

11. Find the integer that is closest to $1000\sum_{n=3}^{10000}\frac{1}{n^2-4}$ (aime)

12. Let "m" be a positive integer, and let $a_0, a_1, \ldots.., a_m$ be a sequence of real numbers such that $a_0 = 37$, $a_1 = 72,\ a_m = 0$ and $a_{k+1} = a_{k-1} - \frac{3}{a_k}$, for k=1,2,3,4,5…. m-1, . Find "m". (aime)

13. A sequence is defined as follows: $a_1 = a_2 = a_3 = 1,$ and , for all positive integers "n", $a_{n+3} = a_{n+2} + a_{n+1} + a_n$. Given that $a_{28} = 6090307$, $a_{29} = 11201821$, and $a_{30} = 20603361$, find the remainder when $\sum_{k=1}^{28} a_k$ is divided by 1000 (aime)